Lab 15: Chapter 11

1. According to a June 2009 report (http://www.alertnet.org/thenews/newsdesk/L31011082.htm), 68% of people with "green" jobs in North America felt that they had job security, whereas 60% of people with green jobs in the United Kingdom felt that they had job security. Suppose that these results were based on samples of 305 people with green jobs from North America and 280 people with green jobs from the United Kingdom

   (a) Use the given information to estimate the difference between the two population proportions.

   $\hat{p}_1 - \hat{p}_2 = 0.68 - 0.60 = 0.08$

   (a) 8%

   (b) Verify that the conditions needed in order for the margin of error formula to be appropriate are met.

   ① Assume the samples are random

   ② There are $X_1 = n_1 \cdot \hat{p}_1 = 207$ successes and $305 - 207 = 98$ failures in sample 1. $X_2 = n_2 \cdot \hat{p}_2 = 168$ There are 168 successes in sample 2 and 112 failures.

   (c) Compute/find the value of the margin of error.

   $$M = 1.96\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

   (c) 7.771%

   (d) Interpret the meaning of the margin of error in the context of this problem.

   It would be unusual for the sample difference, $\hat{p}_1 - \hat{p}_2$, to differ from the actual difference, $p_1 - p_2$, by more than 7.77%

   (e) Construct a 95% confidence interval for the difference between the two population proportions

   (9.8e-4, 0.1564)

   (0.00098, 0.1564)

   (e) (0.098%, 15.64%)

   (f) Communicate the Result: Interpret the confidence interval.

   Assuming that the samples are random or representative of both populations, we are 95% confident that the actual difference in proportions is between 0.098% and 15.64%

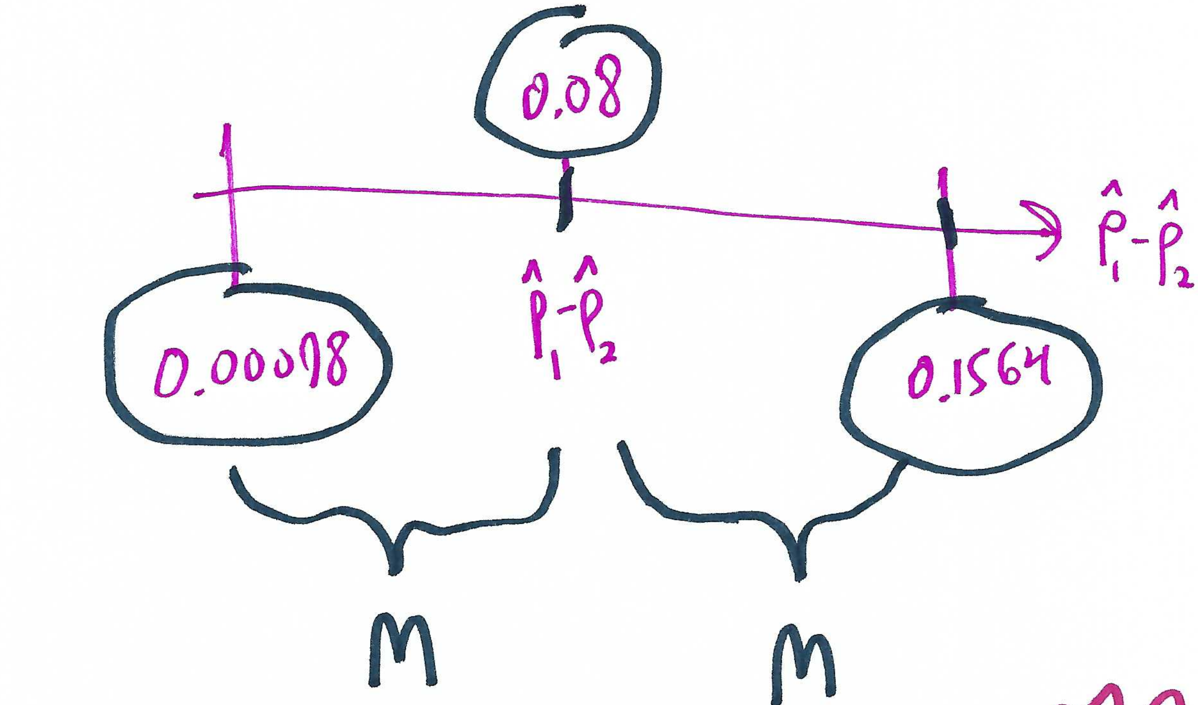
   (g) Communicate the Result: Interpret the confidence *level.*

   The method used to construct the conf. interval estimate is successful in capturing the actual diff. of proportion about 95% of the time

p_1 = the actual % of people in North America with "green" jobs who feel they have job security

p_2 = the actual % of people from the U.K. with "green" jobs who feel they have job security.

$n_1 = 305$	$n_2 = 280$
$\hat{p}_1 = 0.68$	$\hat{p}_2 = 0.60$
x_1 = the number of people in the 1st sample who said "yes"	x_2 = the number of "yes" responses in sample 2
$x_1 = 0.68(305) = n_1(\hat{p}_1) = 207$	$= 0.6(280) = 168$ $(= n_2(\hat{p}_2))$



every conf. interval is $2M$ wide

So,

$$2M = 0.1564 - 0.00098$$

$$M = \frac{0.1564 - 0.00098}{2}$$

$$= 0.07771 \ (7.771\%)$$

$n_1 = n_2 = 100$; $\hat{p}_1 = 0.76$, $\hat{p}_2 = 0.66$

2. A study in the July 7, 2009, issue of USA TODAY stated that the 401(k) participation rate among U.S. employees of Asian heritage is 76%, whereas the participation rate among U.S. employees of Hispanic heritage is 66%. Suppose that these results were based on random samples of 100 U.S. employees from each group.

(a) Use the given information to estimate the difference between the two population proportions

$$\hat{p}_1 - \hat{p}_2 = 0.76 - 0.66 = 0.10$$

(a) 10%

(b) Verify that the conditions needed in order for the margin of error formula to be appropriate are met.

Both samples were random samples.

- $x_1 = n_1 \cdot \hat{p}_1 = 100(0.76) = 76$; $n_1 \cdot (1-\hat{p}_1) = 24$
- $x_2 = n_2 \cdot \hat{p}_2 = 100(0.66) = 66$; $n_2 \cdot (1-\hat{p}_2) = 34$

There are at least 10 successes and failures in each sample.

(c) Compute/find the value of the margin of error. (Use a 99% confidence level)

$$M = 2.58 \cdot \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

Also, $2M = 0.26429 - (-0.0643)$, so $M = 0.32859/2$

(c) 16.4295%

(d) Interpret the meaning of the margin of error in the context of this problem.

It would be unusual for the sample difference, $\hat{p}_1 - \hat{p}_2$, to differ from the actual difference, $p_1 - p_2$, by more than 16%

(e) Construct a 99% confidence interval for the the difference between the two population proportions

$$(-0.0643, 0.26429)$$

(e) (−6.43%, 26.43%)

Since 0% is included within the interval, it is possible that there is no difference between p_1 and p_2.

(f) Communicate the Result: Interpret the confidence interval.

I am 99% confident that the actual difference in proportions is between −6.43% and 26.43%. Also,

(g) Communicate the Result: Interpret the confidence *level*.

The method used to construct the confidence interval estimate is successful in capturing the actual difference between population proportions about 99% of the time.

$\hat{p}_1 = 0.43$, $\hat{p}_2 = 0.25$, $n_1 = n_2 = 200$

3. "Smartest People Often Dumbest About Sunburns" is the headline of an article that appeared in the San Luis Obispo Tribune (July 19, 2006). The article states that "those with a college degree reported a higher incidence of sunburn than those without a high school degree—43% versus 25%." Suppose that these percentages were based on independent random samples of size 200 from each of the two groups of interest (college graduates and those without a high school degree).

(a) Use the given information to estimate the difference between the two population proportions.

$p_1 - p_2 \approx \hat{p}_1 - \hat{p}_2 = 0.43 - 0.25$

(a) 18%

(b) Verify that the conditions needed in order for the margin of error formula to be appropriate are met.

Both samples were random. There are $x_1 = n_1 \cdot \hat{p}_1 = 43$ successes in sample 1 and 200 − 43 = 157 failures. There are $x_2 = n_2 \cdot \hat{p}_2 = 25$ successes in sample 2 and 200 − 25 = 175 failures.

(c) Compute/find the value of the margin of error. (Use a 99% confidence level)

$$M = 2.58\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

Also, $2M = 0.18606 - (-0.0061)$, so $M = 0.19216/2 = 0.09608$

(c) 9.6%

(d) Interpret the meaning of the margin of error in the context of this problem.

It would be unusual for the estimated difference, $\hat{p}_1 - \hat{p}_2$, to differ from $(p_1 - p_2)$ by more than 9.6%.

(e) Construct a 99% confidence interval for the difference between the two population proportions.

(−0.0061, 0.18606)

(e) (−0.61%, 18.606%)

(f) Is zero included in the confidence interval? What does this suggest about the difference in the two population proportions?

Since 0% is in the calculated interval, it is possible that there is no difference between p_1 and p_2.

(g) Communicate the Result: Interpret the confidence interval.

I am 99% confident that the difference in $p_1 - p_2$ is between -0.61% and 18.606%.

(h) Communicate the Result: Interpret the confidence *level.*

A method has been used that is successful in capturing the actual difference in p_1 and p_2 99% of the time.

4. Using the 2% significance level, can you conclude that the proportion of all people with green jobs in North America who feel that they have job security is higher than the corresponding proportion for the United Kingdom?

5. Using the 5% significance level, can you conclude that the 401(k) participation rates are different for all U.S. employees of Asian heritage and all U.S. employees of Hispanic heritage?

(4) Is $p_1 > p_2$? use $\alpha = 0.02$

Step 1

$H_0: p_1 - p_2 = 0$ (there is no diff.)

$H_A: p_1 - p_2 > 0$

Conditions check: (Step 2)

$x_1 = n_1 \cdot \hat{p}_1 = 305(0.68) = \boxed{207}$

and $n_1(1-\hat{p}_1) = 305 - 207 = \boxed{98}$.

Also, $x_2 = n_2 \cdot \hat{p}_2 = 0.6(280) = \boxed{168}$

and $n_2(1-\hat{p}_2) = 280 - 168 = \boxed{112}$.

Also, assume the samples were representative of the two populations.

Step 3 Test statistic

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_c(1-\hat{p}_c)}{n_1} + \frac{\hat{p}_c(1-\hat{p}_c)}{n_2}}}$$

where $\hat{p}_c = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$,

and $z = \boxed{2.07}$

Step 4 P-value and decision

Since we have a right-tailed test, the p-val $= P\left(\hat{p}_1 - \hat{p}_2 > 0.08, \text{ assuming } H_0 \text{ is true}\right)$

$= P(Z > 2.07) \approx \boxed{0.0190}$

Since the p-val $\le \alpha$, reject H_0.

Step 5 There is convincing sample evidence that suggests that $p_1 > p_2$.

(5) Is $p_1 \neq p_2$? Use $\alpha = 0.05$

$\hat{p}_1 - \hat{p}_2 = 0.10$

$x_1 = 76$

$n_1 = 100$

$x_2 = 66$

$n_2 = 100$

Step 1 $H_0: p_1 - p_2 = 0$ (no difference)

$H_A: p_1 - p_2 \neq 0$ (there is a difference between p_1 and p_2.)

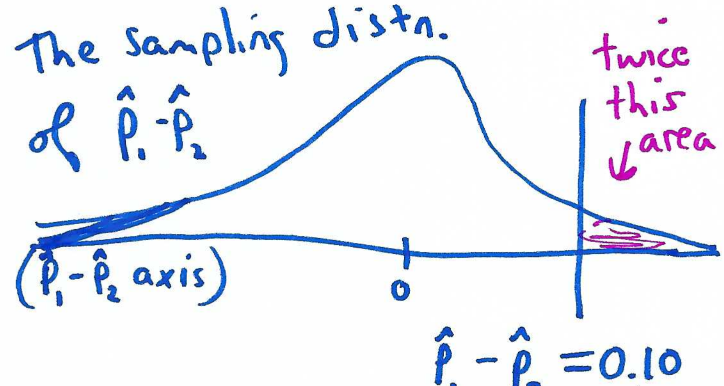
Step 2 Conditions Check

The conditions were already checked in problem 2b.

Step 3 Test statistic

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_c(1-\hat{p}_c)}{n_1} + \frac{\hat{p}_c(1-\hat{p}_c)}{n_2}}}, \text{ where } \hat{p}_c = \frac{x_1 + x_2}{n_1 + n_2},$$

and $Z = \boxed{1.56}$



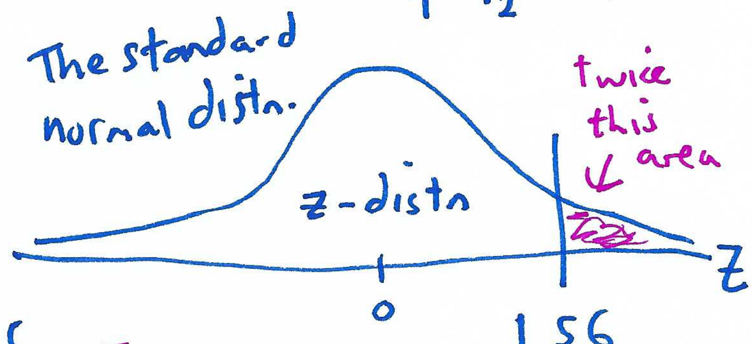
Step 4 P-value and decision

Since we have a 2-tailed test, the p-value equals

$$2 \cdot P(\hat{p}_1 - \hat{p}_2 > 0.10)$$

$$= 2 \cdot P(Z > 1.56)$$

$$\approx \boxed{0.1192}$$



Since the p-val > α, fail to reject H_0

Step 5 There is not convincing sample evidence to suggest that there is a difference between p_1 and p_2.